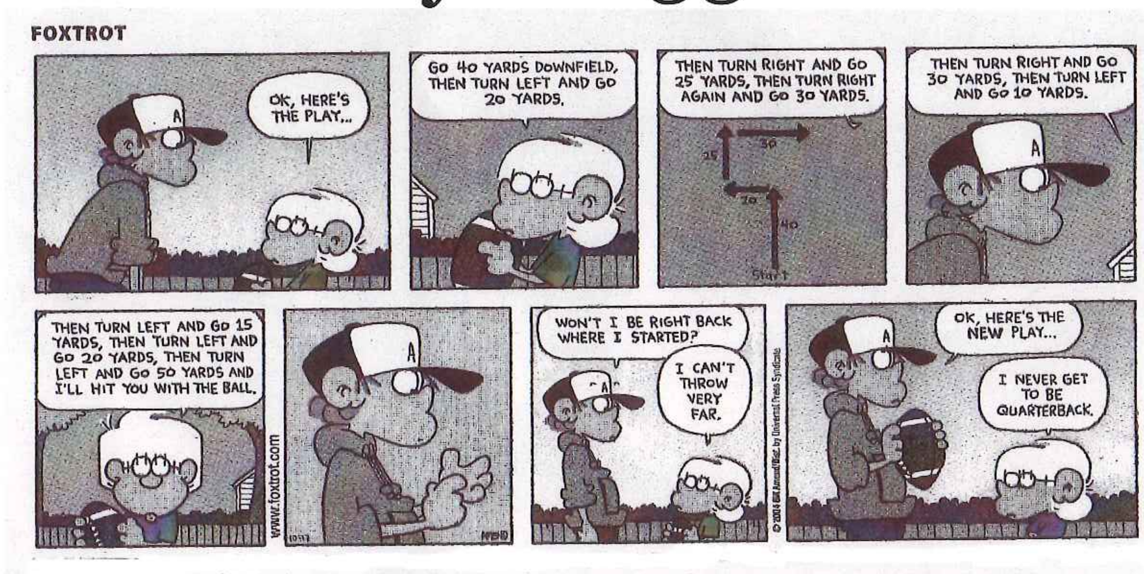
Physics 3310



Notes and Problem Sets
Kinematics in One Dimension

Chapter 2

Giancoli

NOTES
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STUDENT NAME

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TEACHER'S NAME

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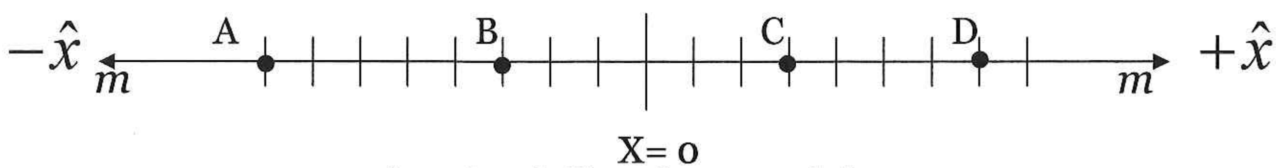
PERIOD

Lesson 1: Position, Displacement and Distance

☐ **Position is always defined from the origin and always has $+\hat{x}$ or $-\hat{x}$ listed with the position... unless you are at the origin and then your position is just x = 0.**



List the following Positions

$\vec{x}_A = -8m\,\hat{x}$

$\vec{x}_B = -3m\,\hat{x}$

$\vec{x}_C = 3m\,\hat{x}$

$\vec{x}_D = 8m\,\hat{x}$

☐ **Note: An object can have Positive +, Negative –, or "Zero" position**

ALL DEPENDS ON WHERE YOU PUT YOUR ZERO.

ALL RELATIVE TO YOUR FRAME OF REFERENCE

☐ Definition of Displacement

Δ = DELTA; "CHANGE"

$$\boxed{\Delta\vec{x} \equiv \vec{x}_f - \vec{x}_i}$$

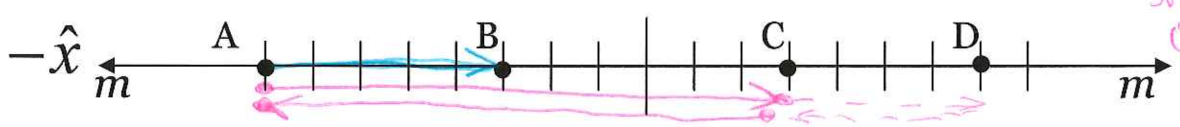
☐ **Displacement always has $+\hat{x}$ or $-\hat{x}$ listed with the Displacement… unless your position did not change and then your displacement is just $\Delta\vec{x} = 0$**

DISPLACEMENT IS A VECTOR!

☐ **Distance is the total "ground" covered.**

DISTANCE IS A SCALAR

THINK OF A TRACK RUNNER
1 mile = 4 laps
↑ DISTANCE
DISPLACEMENT IS 0 – START AND FINISH AT SAME SPOT.
OTHER EXAMPLES?



$\Delta\vec{x}_{A\to B} = \vec{x}_B - \vec{x}_A = -3m + (+8m) = 5m\hat{x}$

Distance = 5m

$\Delta\vec{x}_{A\to D\to C} = \vec{x}_C - \vec{x}_A = +8m + (+8m) = 11m\hat{x}$

Distance = 8m + 8m + 5m = 21m

$\Delta\vec{x}_{C\to A} = \vec{x}_A - \vec{x}_C = -8m + 3m = -11m\hat{x}$

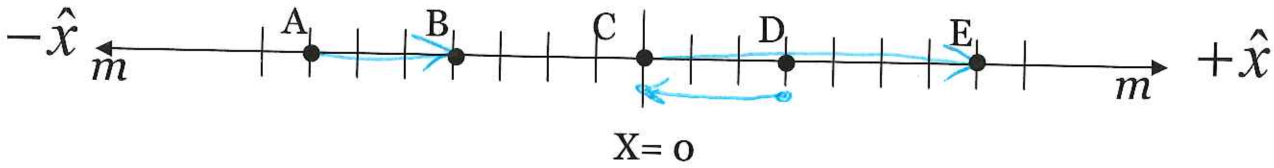
Distance= 3m + 8m = 11m

SAME DISPLACEMENTS BUT OPPOSITE DIRECTIONS

$\Delta\vec{x}_{D\to B\to C\to D} = 0m$

Distance = 4m + 6m + 6m + 4m = 20m

Lesson 1 Problems: Position, Displacement and Distance



1. For each of the points above list the positions.

$\vec{x}_A = -7m\hat{x}$ $\vec{x}_B = -4m\hat{x}$ $\vec{x}_C = 0m$ $\vec{x}_D = 3m\hat{x}$ $\vec{x}_E = 7m\hat{x}$

2. Find the displacement and the distance traveled in each case below.

$\Delta\vec{x}_{A\to D\to B} = x_B - x_A = -4m + (+7m) = \boxed{3m\hat{x}}$

Distance = $3m + 4m + 3m + 3m + 4m = \boxed{17m}$

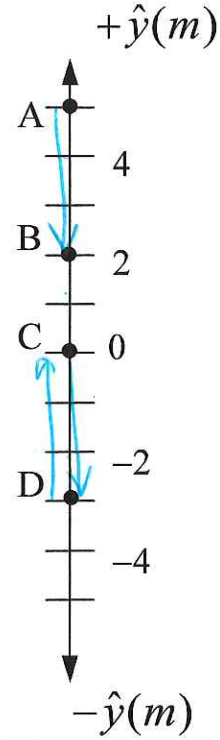
$\Delta\vec{x}_{C\to B\to E} = x_E - x_C = 7m - 0 = \boxed{7m\hat{x}}$

Distance = $4m + 7m + 4m = \boxed{15m}$

$\Delta\vec{x}_{D\to A\to C} = x_C - x_D = 0m - 3m = \boxed{-3m\hat{x}}$

Distance = $10m + 7 = \boxed{17m}$

3. For each of the points on the y-axis list the positions.



$\vec{y}_A =$ $5m\,\hat{y}$

$\vec{y}_B =$ $2m\,\hat{y}$

$\vec{y}_C =$ $0m\,\hat{y}$

$\vec{y}_D =$ $-3m\,\hat{y}$

4. Find the displacement and the distance traveled in each case below.

$\Delta\vec{y}_{A\to D\to B} =$ $y_B - y_A = 2m - 5m = -3m\,\hat{y}$

Distance = $8m + 5m = 13m$

$\Delta\vec{y}_{C\to B\to D} =$ $y_D - y_C = -3m\hat{y} - 0m\hat{y} = -3m\hat{y}$

Distance = $7m$

$\Delta\vec{y}_{D\to A\to C} =$ $y_C - y_D = 0m\hat{y} + (+3\,m\hat{y}) = 3m\hat{y}$

Distance = $13m$

OPPOSITE DISPLACE-MENTS

Lesson 2. Speed vs Velocity

☐ Definition of Average Speed, v

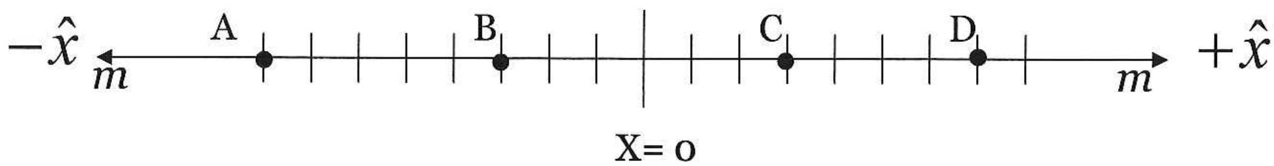
SPEED IS A SCALAR

EQUALITY BY DEFINITION

$$\boxed{v \equiv \frac{d}{t} = \frac{\text{distance}}{time} = speed}$$

☐ **Speed tells you how fast an object is moving. It does not tell you in which direction. There is no such thing as negative speed. The least amount of speed an object can ever have is zero, the object is at rest.** UNITS = METERS/second

Recall from the last example:



$$\Delta\vec{x}_{D\to B\to C\to D} = 0\,m\,\hat{x}$$

Distance = 20 m

If the particle traveled this distance in 4 seconds, what was the average speed of the particle for the round trip?

$$\text{SPEED} = v = \frac{d}{t} = \frac{20\,m}{4\,s} = 5\,m/s$$

☆ WHAT ABOUT 2 SECONDS?

$$v = \frac{d}{t} = \frac{20\,m}{2\,s} = 10\,m/s$$

$\therefore$ 1/2 TIME → 2 TIMES SPEED

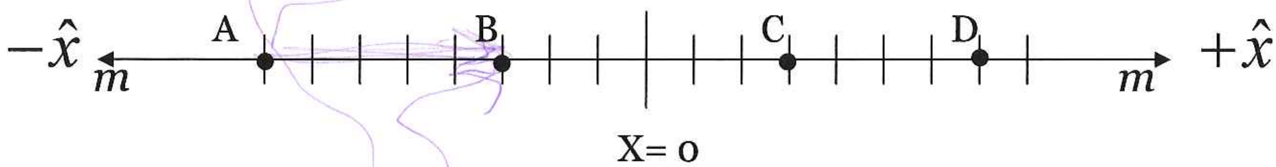
☐ Definition of Average Velocity

$$\boxed{\vec{v} \equiv \frac{\Delta\vec{x}}{\Delta t} = \frac{\text{displacement}}{\textit{time}}}$$

VECTOR!

HOW MUCH IT DRIFTS DOESN'T MATTER WHAT IT DOES, JUST HOW FAST IT GOES FROM a TO b

☐ **Average Velocity always has the direction $+\hat{x}$ or $-\hat{x}$ listed with the answer… unless your average velocity is zero and then $\vec{v} = 0$ = stop**



A particle takes 10 seconds to follow the paths given below?

$\Delta\vec{x}_{A\to C\to B\to D} = \cancel{27m}\ x_D - x_A = 15m\,\hat{x}$

Distance = 27m

$\Delta\vec{x}_{A\to D\to A} = x_A - x_A = 0$

Distance = $(15m)2 = 30m$

Calculate the average velocity of the particle.

$\vec{v}_{A\to C\to B\to D} = \frac{\Delta\vec{x}}{\Delta t} = \frac{15m\,\hat{x}}{10s} = \frac{2}{3}\,m/s\,\hat{x}$

$\vec{v}_{A\to D\to A} = \frac{0}{10s} = 0\,m/s$

Calculate the average speed of the particle as it travels from A to D.

$v_{A\to C\to B\to D} = \frac{d}{t} = \frac{27m}{10s} =$

$v_{A\to D\to A} = \frac{30m}{10s} = 3\,m/s$

MEASURE SPEED AND VELOCITY OF STUDENT AROUND ROOM

SPEED → "HOW FAST AN OBJECT IS MOVING"

VELOCITY → "THE RATE AT WHICH AN OBJECT CHANGES ITS POSITION"

Lesson 2 Problems: Speed vs Velocity

1. Light from the Sun reaches the Earth in 8.3 min. The speed of light is $3.0\text{x}10^8$ m/s. How far is the Sun from the Earth?

$8.3\,mins \times \frac{60\,sec}{1\,min} = 498\,sec$

$V = \frac{d}{t}$

$3\times10^8\,m/s = \frac{d}{498\,sec} \Rightarrow \boxed{d = 1.494\times10^{11}\,m}$

Book → $149.6\times10^6\,Km = 149.6\times10^9\,m$
$= 1.496\times10^{11}\,m$ ✓

2. What must be you average speed in order to travel 230 km in 3.25 h?

$V = d/t$

$V = \frac{230\,km}{3.25\,h} = \boxed{70.769\,Km/hr}$

3. If you are driving 110 km/h along a straight road and you look to the side for 2.0 s, how far do you travel during this inattentive period?

$\frac{110\,Km}{h} \times \frac{1\,hr}{3600\,s} \times \frac{1000\,m}{1\,km} = 30.\overline{555}\,m/s$

$V = d/t$

$30.\overline{555}\,m/s = d/2s$

$\boxed{\therefore d = 61.\overline{11}\,m}$

4. You are driving home from school steadily at 65 mph for 130 miles. It then begins to rain and you slow to 55 mph. You arrive home after driving 3 hours and 20 minutes. (a) How far is your hometown from school? (b) What was your average speed?

a. $v = d/t$

$65mph = \frac{130 miles}{t}$

$t = 2hrs$

$3\frac{1}{3}hr - 2hr = 1\frac{1}{3}hr$

$v = d/t$

$55 = d/1\frac{1}{3}$

$d = 73.33$ miles

TOTAL DISTANCE $= \boxed{203.\overline{33} \text{ miles}}$

b. $v = \frac{d}{t}$

$v = \frac{203.\overline{33} \text{ miles}}{3\frac{1}{3} hr}$

$\boxed{v = 61 mph}$

5. An airplane travels 2100 km at a speed of 800 km/h, and then encounters a tailwind that boosts its speed to 1000 km/h for the next 1800 km. What was the total time for the trip? What was the average speed of the plane for this trip?

$v = d/t$

$800 km/h = \frac{2100 km}{t}$

$t = 2.625 hr$

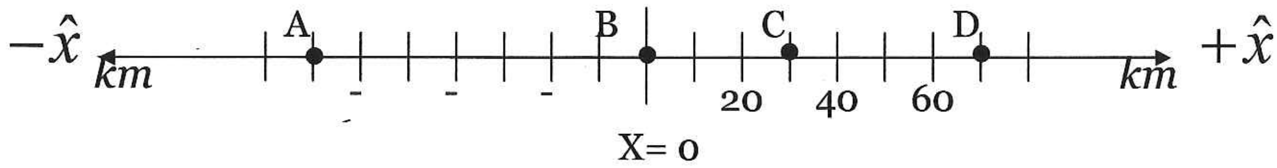
$1000 km/h = \frac{1800 km}{t}$

$t = 1.8 hr$

$2.625hr + 1.8hr = \boxed{4.425hr}$

$v = d/t$

$= \frac{2100 km + 1800 km}{4.425 hr} = \boxed{881.356 \, km/hr}$



6. A car starts at town D and drives to town A arriving in 1.5 hrs.
   a.) Calculate the average speed of the car.
   b.) Calculate the average velocity of the car.

a. $V = \frac{d}{t} = \frac{70Km + 70Km}{1.5hr} = 93.\overline{33}\ Km/hr$

b. $\bar{V} = \frac{\Delta x}{t} = \frac{-140Km\ \hat{x}}{1.5hr} = -93.\overline{33}\ Km/hr\ \hat{x}$

7. A car starts at town D and drives to town A arriving in 1.5 hrs (same as above). It then continues on to town C and arriving 1 hr. after leaving town A.
   a.) Calculate the average speed of the car.
   b.) Calculate the average velocity of the car.

a. $V = \frac{d}{t} = \frac{140km + 100Km}{1.5hr + 1hr} = \boxed{96\ Km/hr}$

b. $\bar{V} = \frac{\Delta x}{t} = \frac{40Km}{1.5hr + 1hr} = \boxed{16\ Km/hr}$

Concept Questions for Lesson 2. – Speed and Velocity

1. Does a car speedometer measure speed, velocity or both?

SPEED → NO DIRECTION

2. Can an object have a varying velocity if its speed is constant? If yes, give examples.

YES, VELOCITY IS A VECTOR SO DIRECTION CAN CHANGE WITH CONSTANT SPEED. ex: FERRIS WHEEL

3. Can an object have a varying speed if its velocity is constant? If yes, give examples.

NO, VELOCITY IS SPEED AND DIRECTION, SO CONSTANT VELOCITY NEEDS CONSTANT SPEED

4. When an object moves with constant velocity, does its average velocity during any time interval differ from its instantaneous velocity at any instant?

NO. CONSTANT VELOCITY IMPLIES THE SAME VELOCITY SO SINCE IT DOESN'T CHANGE, THE AVERAGE AND INSTANTANEOUS MUST BE THE SAME.

5. In drag racing, is it possible for the car with the greatest speed crossing the finish line to lose the race? Explain

YES. THE CAR MIGHT NOT ACCELERATE AS QUICKLY SO IT TAKES LONGER TO ACHIEVE A FASTER SPEED.

6. Can an object have zero position and non-zero speed?

YES. PASSING THROUGH THE ZERO POSITION FOR A MOMENT

Lesson 3. Acceleration

- ☐ **Average Acceleration is defined by:**

$$\boxed{\vec{a} \equiv \frac{\Delta \vec{v}}{\Delta t} = \frac{\textit{Change in Velocity}}{\textit{Change in Time}}}$$

SINCE VELOCITY IS A $\boxed{\text{VECTOR}}$ AND TIME IS A SCALAR, a = VECTOR

SINCE $\frac{\text{VECTOR}}{\text{SCALAR}}$ = VECTOR

Units: $\bar{a}_{ave} = \frac{m/s}{s} = \frac{m}{s^2}$ = $\frac{m/s}{s}$

METERS PER SECOND SQUARED

METERS PER SECOND PER SECOND

- ☐ **Average Acceleration always has the direction $+\hat{x}$ or $-\hat{x}$ listed with the answer… unless your average acceleration is zero and then $\vec{a} = 0$**

- ☐ **Think of the Average Acceleration as "pulling" on the velocity of an object. To understand the effect of this acceleration you MUST know the initial velocity of the object that is undergoing acceleration.**

$a = \frac{v_f - v_i}{\Delta t}$

Positive Acceleration

☐ Ex. $\vec{a} = +10\frac{m}{s^2}\hat{x}$ this means that the velocity is changing 10 m/s every second in the + x direction.

☐ Ex. At t = 0 let $\vec{v}_i = +20\frac{m}{s}\hat{x}$ and $\vec{a} = +10\frac{m}{s^2}\hat{x}$.
This means that the velocity is changing 10 m/s every second in the + x direction. $20 m/s\,\hat{x} + 10 m/s\,\hat{x} = 30 m/s$

At t = 1 second v = $30 m/s\,\hat{x}$ $\xrightarrow{}$

At t = 2 seconds v = $40 m/s\,\hat{x}$ $\xrightarrow{40 m/s}$ $\xrightarrow{\text{ACCELERATION}}$

At t = 3 seconds v = $50 m/s\,\hat{x}$ $\xrightarrow{}$

☐ Ex. Take $\vec{v}_i = -20\frac{m}{s}\hat{x}$ and $\vec{a} = +10\frac{m}{s^2}\hat{x}$. This means that the velocity is changing 10 m/s every second in the + x direction. v_i: $\xleftarrow{-20 m/s}$ a: $\xrightarrow{10 m/s^2}$

At t = 1 second v = $-10 m/s\,\hat{x}$ $\xleftarrow{-10 m/s}$ a. $a = \frac{v_f - v_i}{t}$

At t = 2 seconds v = $0 m/s\,\hat{x}$ $\cdot$ 0 m/s $10 m/s^2 = \frac{v_f + (-20 m/s)}{1}$

At t = 3 seconds v = $10 m/s\,\hat{x}$ $\xrightarrow{10 m/s}$ $v_f = -10 m/s$

At t = 4 seconds v = $20 m/s\,\hat{x}$ $\xrightarrow{20 m/s}$ b. $10 m/s^2 = \frac{v_f + (+10 m/s)}{1}$

$v_f = 0$

MATH WAY

-OR-

$10 m/s = \frac{v_f + (+20 m/s)}{2}$

$v_f = 0$

Negative Acceleration

☐ Ex. $\vec{a} = -10\frac{m}{s^2}\hat{x}$ this means that the velocity is changing 10 m/s every second in the – x direction.

☐ Ex. At t = 0 sec let $\vec{v}_i = +20\frac{m}{s}\hat{x}$ and $\vec{a} = -10\frac{m}{s^2}\hat{x}$.
This means that the velocity is changing 10 m/s every second in the – x direction. $\xrightarrow{20 m/s}$ $a = -10 m/s^2\,\hat{x}$ $\longleftarrow$

At t = 1 second v = $10 m/s\,\hat{x}$ $t=1$: $-10 m/s^2 = \frac{v_f - 20 m/s}{1} \Rightarrow v_f = 10 m/s$

At t = 2 seconds v = $0 m/s\,\hat{x}$ $t=2$: $-10 m/s^2 = \frac{v_f - 10 m/s}{1} \Rightarrow v_f = 0 m/s$

At t = 3 seconds v = $-10 m/s\,\hat{x}$ $t=3$: $-10 m/s^2 = \frac{v_f - 0 m/s}{1} \Rightarrow v_f = -10 m/s$

AT t = 4 SECONDS V = $-20 m/s\,\hat{x}$ $t=4$: $-10 m/s^2 = \frac{v_f + (+10 m/s)}{1} \Rightarrow v_f = -20 m/s$

~~At t = 4 seconds v =~~

☐ Ex. Take $\vec{v}_i = -20\frac{m}{s}\hat{x}$ and $\vec{a} = -10\frac{m}{s^2}\hat{x}$. This means that the velocity is changing 10 m/s every second in the – x direction.

At t = 1 second v = $-30\,m/s\,\hat{x}$ $\quad -10\,m/s^2 = \frac{v_f + (+20\,m/s)}{1} \Rightarrow v_f = -30\,m/s$

At t = 2 seconds v = $-40\,m/s\,\hat{x}$ $\quad -10\,m/s^2 = \frac{v_f + (+30\,m/s)}{1} \Rightarrow v_f = -40\,m/s$

At t = 3 seconds v = $-50\,m/s\,\hat{x}$ $\quad -10\,m/s^2 = \frac{v_f + (+40\,m/s)}{1} \Rightarrow v_f = -50\,m/s$

OTHER WAYS TO SOLVE! USE $v_i = -20\,m/s$ EACH TIME, CHANGE Δt

Lesson 3 Problems: Acceleration

1. An object moving to the right with an initial speed of 25 m/s suddenly feels a constant acceleration of 5 m/s^2 to the left $\quad 25\,m/s \rightarrow \quad \leftarrow 5\,m/s^2$
   a.) Write down the initial velocity and the acceleration.
   $v_i = 25\,m/s\,\hat{x}$
   $a = -5\,m/s^2\,\hat{x}$
   b.) 3 seconds after the acceleration acts on the object what is the objects velocity?
   $a = \frac{v_f - v_i}{t} \Rightarrow -5\,m/s^2 = \frac{v_f - (25\,m/s)}{3s} \quad \therefore \boxed{v_f = 10\,m/s\,\hat{x}}$
   c.) How many seconds does it take to come to a stop?
   $-5\,m/s^2 = \frac{0 - 25\,m/s}{t} \Rightarrow +5t = +25 \quad \therefore \boxed{t = 5 sec}$
   d.) What is the velocity of the object 2 seconds after it stopped?
   $-5\,m/s^2 = \frac{v_f - 25\,m/s}{7 sec} \quad \therefore \boxed{v_f = -10\,m/s\,\hat{x}}$
   $-35\,m/s = v_f - 25\,m/s$
2. An object moving to the left with an initial speed of 5 m/s suddenly feels a constant acceleration of 10 m/s^2 to the left $\quad \leftarrow 5\,m/s \quad \leftarrow a = 10\,m/s$
   e.) Write down the initial velocity and the acceleration
   $v_i = -5\,m/s\,\hat{x}$
   $a = -10\,m/s^2\,\hat{x}$
   f.) 3 seconds after the acceleration acts on the object what is the objects velocity?
   $-10\,m/s^2 = \frac{v_f + (+5\,m/s)}{3 sec} \quad \therefore \boxed{v_f = -35\,m/s\,\hat{x}}$
   $-30\,m/s = v_f + 5\,m/s$

g.) How many seconds does it take to come to a stop or does it stop?

IT DOES NOT STOP.

3. An object passes through the origin moving upward with an initial speed of 50 m/s and is acted on by a downward acceleration of 10 m/s^2.

   h.) How many seconds does it take the object to come to a stop.

   $v_i = 50\, m/s\, \hat{y}$ $\qquad -10\, m/s^2 = \frac{0 - 50\, m/s}{t}$

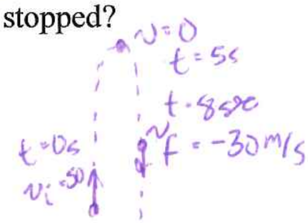
   $a = -10\, m/s^2\, \hat{y}$ $\qquad -10t = -50$ $\qquad \therefore \boxed{t = 5 sec}$

   i.) What is the speed 8 seconds after it has stopped?

   $-10\, m/s^2 = \frac{v_f - 0\, m/s}{8 sec}$

   $-80\, m/s = v_f - 0\, m/s$

   $\boxed{\vec{v}_f = -80\, m/s\, \hat{y}}$



Concept Questions for Lesson 3. – Acceleration

1. If one object has a greater speed than a second object. Does the first necessarily have a greater acceleration? Explain, using examples

   NO. A CAR CAN BE TRAVELLING AT A CONSTANT 50 mph BUT A CAR ACCELERATING FROM 0 TO 10 mph HAS A GREATER ACCELERATION.

2. Can an object have a northward velocity and a southward acceleration? Explain

   YES. A CAR CAN BE TRAVELING NORTH BUT APPLYING THE BREAKS SO IT IS ACCELERATING IN THE SOUTHWARD DIRECTION.

3. Can the velocity of an object be negative when its acceleration is positive? What about vice versa?

   Yes. TRAVELING LEFT (NEGATIVE v) ← AND SLOWING DOWN (POSITIVE a) →.

   Yes. TRAVELING RIGHT → (POSITIVE V) AND SLOWING DOWN ← (NEGATIVE a)

4. Give examples where both the velocity and acceleration are negative.

   SPEEDING UP IN THE NEGATIVE DIRECTION,

   VELOCITY ← AND ACCELERATION ←

5. Is it possible for an object to have a negative acceleration while increasing in speed? If so, provide an example

YES IF IT IS TRAVELING IN THE NEGATIVE DIRECTION. MOVING LEFT ~~RIGHT~~ AND ~~ACC~~ NEGATIVE ACCELERATION WILL INCREASE SPEED.

6. Can an object be increasing in speed as its acceleration decreases? If so, give an example, If not, explain

YES, IT JUST INCREASES SPEED AT A SLOWER RATE.

A CAR CAN START OFF WITH A RAPID ACCELERATION BUT SLOWLY COME TO ITS FINAL SPEED.

$t=0$	$t=1$	$t=2$	$t=3$
$v=0$	$v=20$	$v=30$	$v=35$
$a=20$	$a=10$	$a=5$	$a=0$

7. As a freely falling object speeds up, what is happening to its acceleration due to gravity? Does it increase, decrease, or stay the same

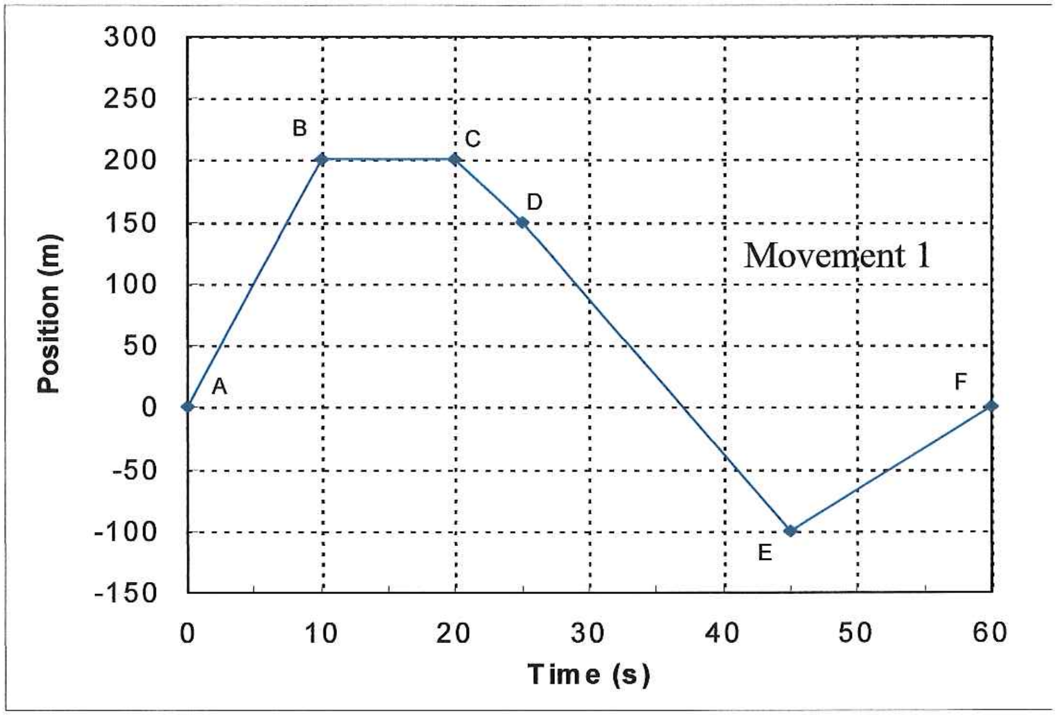
STAY THE SAME. ALWAYS $-9.8\,m/s^2$ g

Lesson 4. Position vs. Time Graphs

Position vs Time graphs are "slope pictures" of the velocity equation

$$\boxed{\vec{v} \equiv \frac{\Delta\vec{x}}{\Delta t} = \frac{\text{y-axis}}{\text{x-axis}} = \frac{rise}{run} = slope}$$

Position vs. Time Graph for a Complete Trip



x	y
Time (s)	Position (m)
0	0
10	200
20	200
25	150
45	-100
60	0

Find the average velocity as the object moves from: $v = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{\Delta t}$

a.) A to B

$v = \frac{200-0m}{10s} = \boxed{20\ m/s}$

b.) B to C

$v = \frac{200m-200m}{10s} = \boxed{0\ m/s}$

c.) C to D

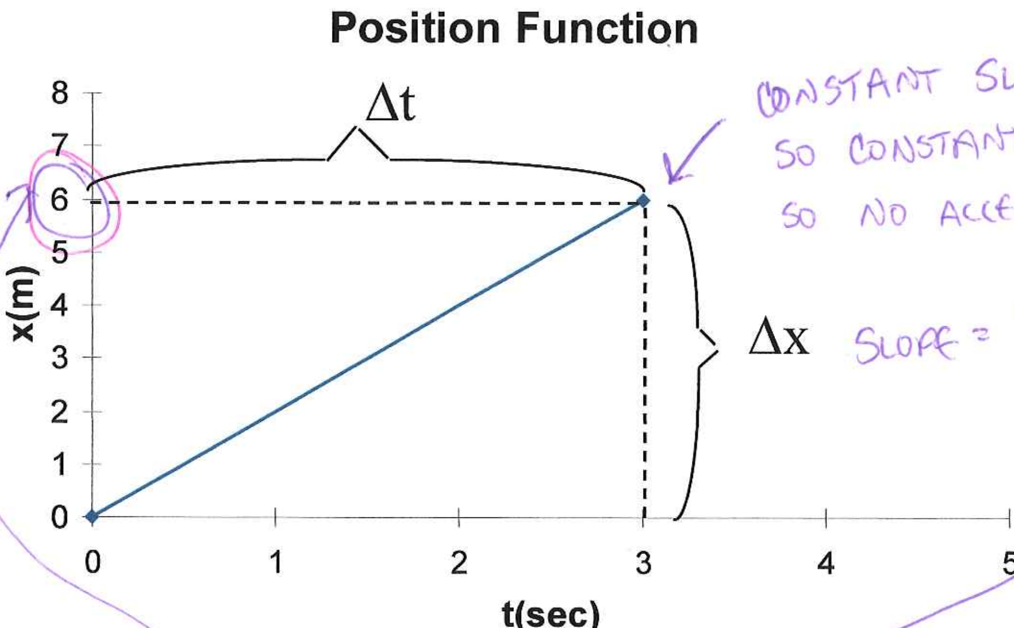
$v = \frac{150m-200m}{5s} = \frac{-50m}{5s} = \boxed{-10\ m/s}$

d.) A to E

$v = \frac{-100m-0}{45s} = \boxed{-\frac{20}{9}\ m/s}$

e.) A to F $v = \frac{0m-0m}{60s} = \boxed{0\ m/s}$

Going from (x vs t) to (v vs t)



CONSTANT SLOPE,
SO CONSTANT VELOCITY
SO NO ACCELERATION

SLOPE = $\frac{6m}{3s}$ = 2 m/s

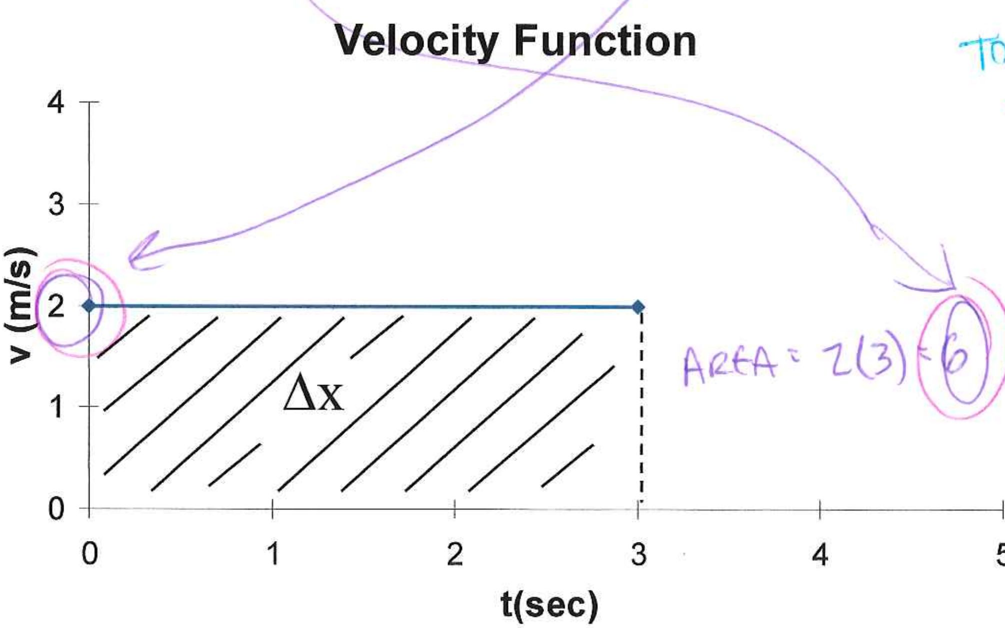
$$v_{ave} = \frac{\Delta x}{\Delta t}$$

DERIVATIVE OF x VS t
GIVES v VS t

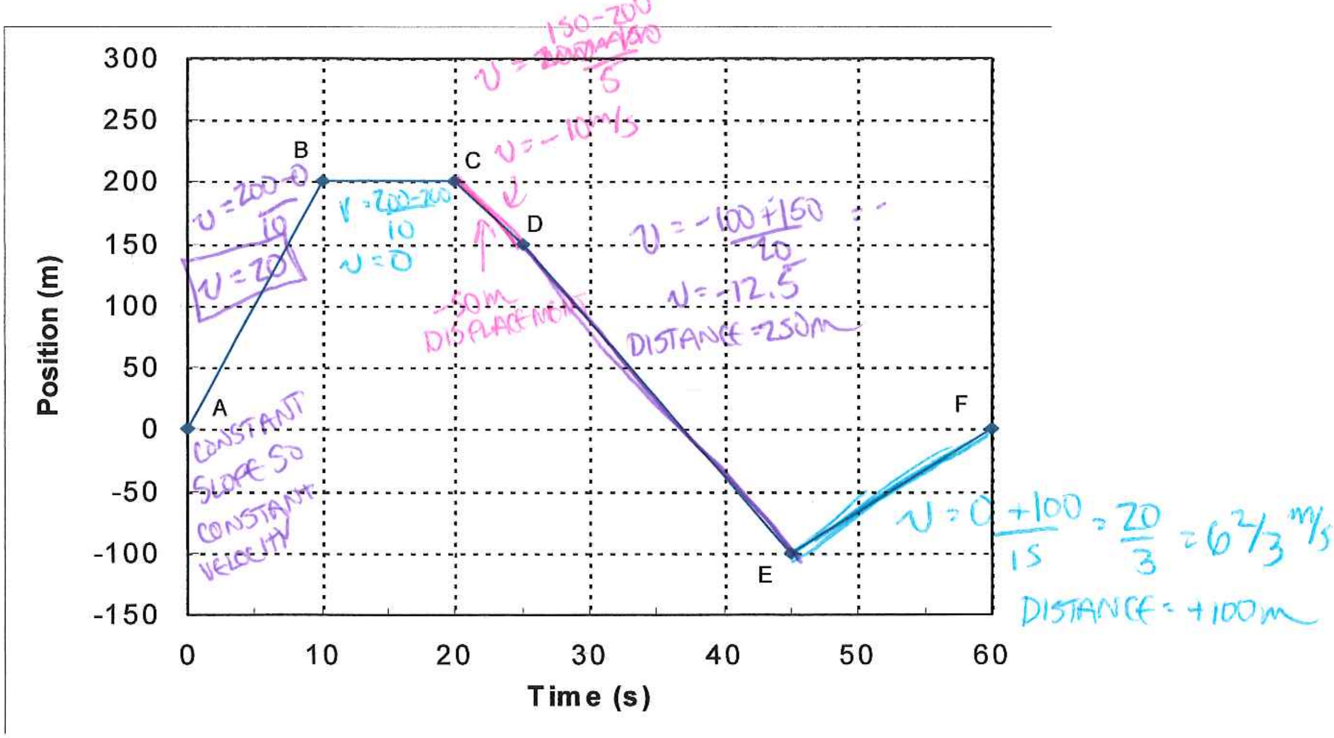
Area!

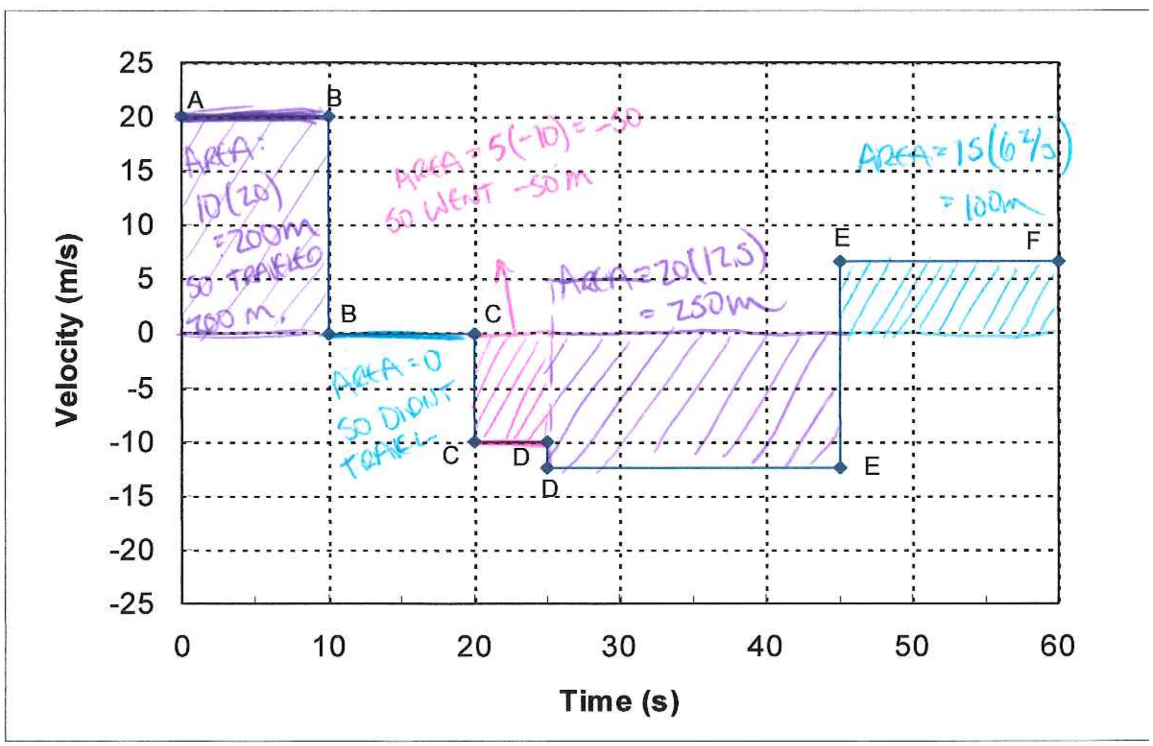
INTERGRAL OF v VS t
GIVES x VS t

TO FIND v VS t,
TAKE SLOPE OF x VS t

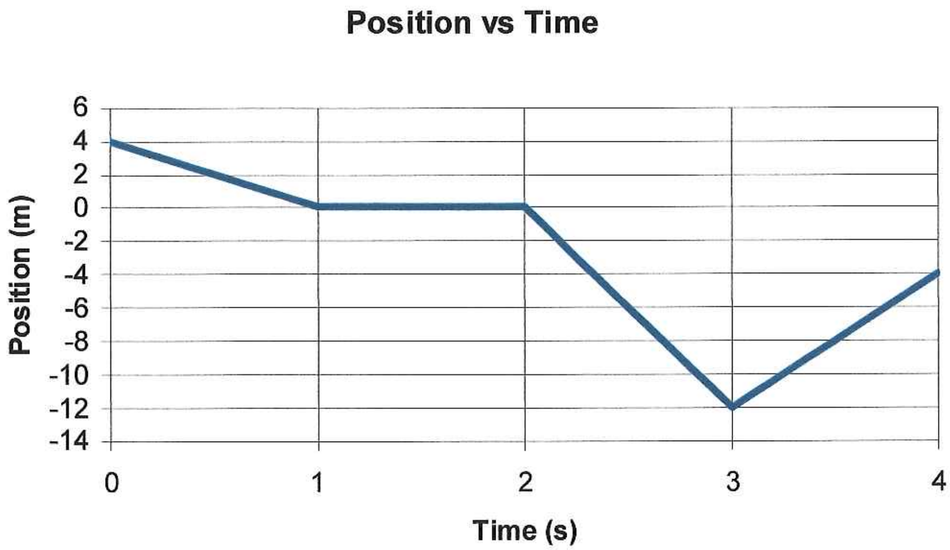


Velocity vs. Time Graph for a Complete Trip





Lesson 4 Problems: Position vs Time Graphs



1. What total distance does the object cover over the time interval 0 < t < 4s.

DISTANCE = ~~AREA~~

$4m + 0m + 12m + 8m = \boxed{24m}$

2. What is the total displacement of the object over the time interval 0 < t < 4s

~~AAAA~~ $\Delta x = x_f - x_i$

$= -4m + -4m$ $\therefore$ $\boxed{\Delta x = -8m\hat{x}}$

3. What is the average speed of the object over the time interval 0 < t < 4 s.

$v = d/t = 24m/4s$ $\boxed{v = 6 m/s}$

4. Calculate the average velocity of the object over each of the time intervals above.

$0<t<1: \vec{v} = \frac{0m-4m}{1s} = \boxed{-4 m/s\,\hat{x}}$

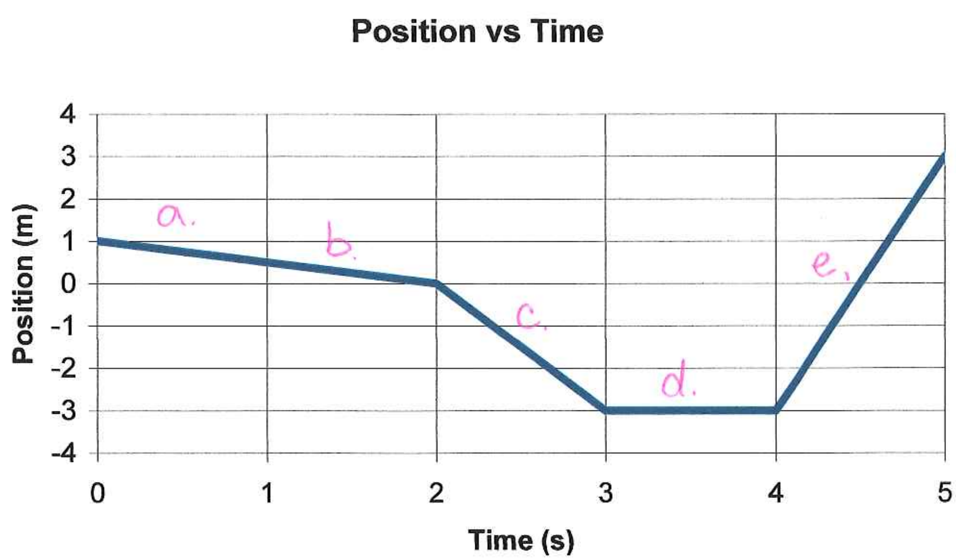
$2<t<3: \bar{v} = \frac{-12m-0m}{1s} = \boxed{-12 m/s\,\hat{x}}$

$1<t<2: \bar{v} = \frac{0m-0m}{1s} = \boxed{0 m/s}$

$3<t<4: \bar{v} = \frac{-4m+(+12m)}{1s} = \boxed{8 m/s\,\hat{x}}$

5. What is the average velocity of the object over the time interval 0 < t < 4 s.

$\bar{v} = \frac{\Delta x}{t} = \frac{-8m\hat{x}}{4} = \boxed{-2 m/s\,\hat{x}}$



1. What total distance does the object cover over the time interval 0 < t < 5s.

$d = .5m + .5m + 3m + 0m + 6m$

$\boxed{d = 10m}$

2. What is the total displacement of the object over the time interval 0 < t < 5s

$\Delta x = x_f - x_i$
$= 3m - 1m$

$\therefore \boxed{\Delta x = 2m\hat{x}}$

3. What is the average speed of the object over the time interval 0 < t < 5s.

$v = d/t = \frac{10m}{5s}$ $\therefore \boxed{v = 2m/s}$

4. Calculate the average velocity of the object over each of the time intervals above.

a. $\bar{v} = \frac{.5m - 1m}{1s} = \boxed{-.5m/s\hat{x}}$

b. $\bar{v} = \frac{0m - .5m}{1s} = \boxed{-.5m/s\hat{x}}$

c. $\bar{v} = \frac{-3-0}{1} = \boxed{-3m/s\hat{x}}$

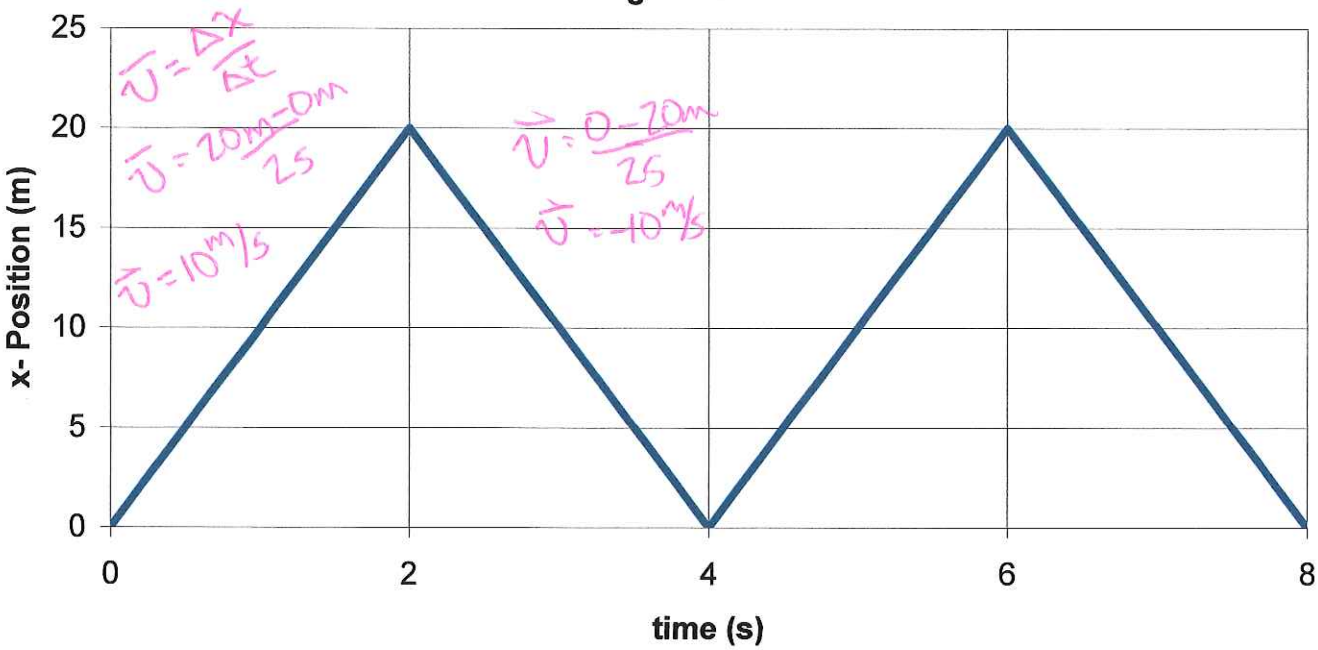
d. $\bar{v} = \frac{-3+(+3)}{1} = \boxed{0m/s}$

e. $\bar{v} = \frac{3m-(-3m)}{1} = \boxed{6m/s}$

5. What is the average velocity of the object over the time interval 0 < t < 5s.

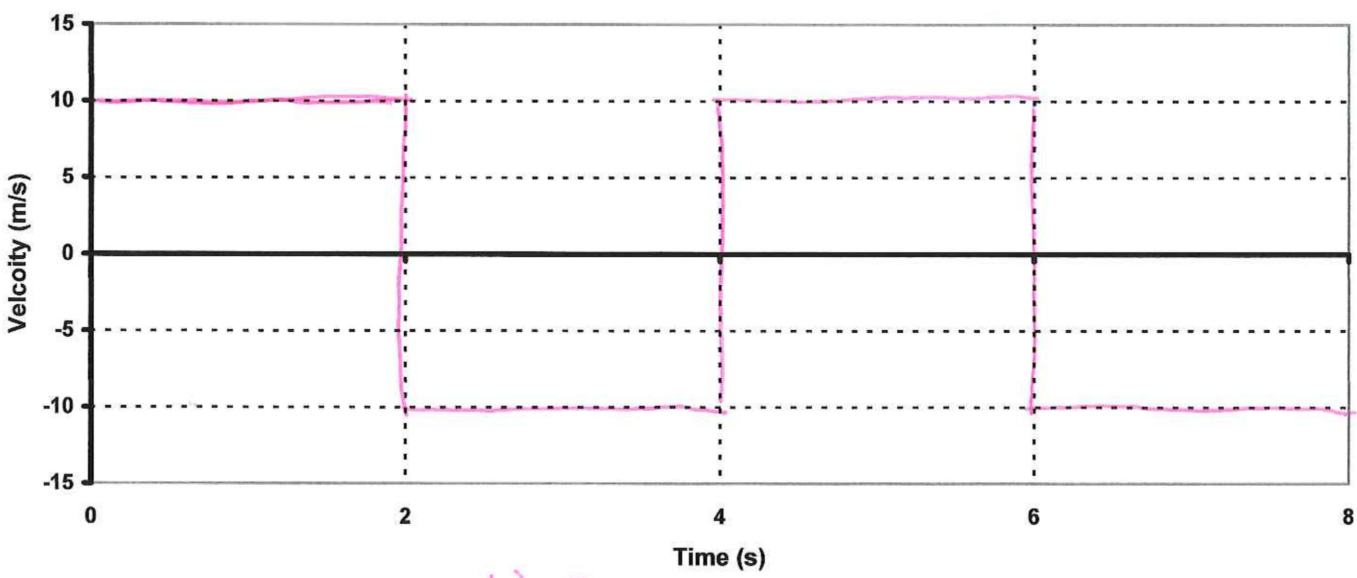
$\bar{v} = \frac{2m}{5s} = \boxed{.4m/s\hat{x}}$

Figure 2



4. In Figure 2b sketch the velocity function for a motion in which the position function is given by Figure 2

Figure 2 b

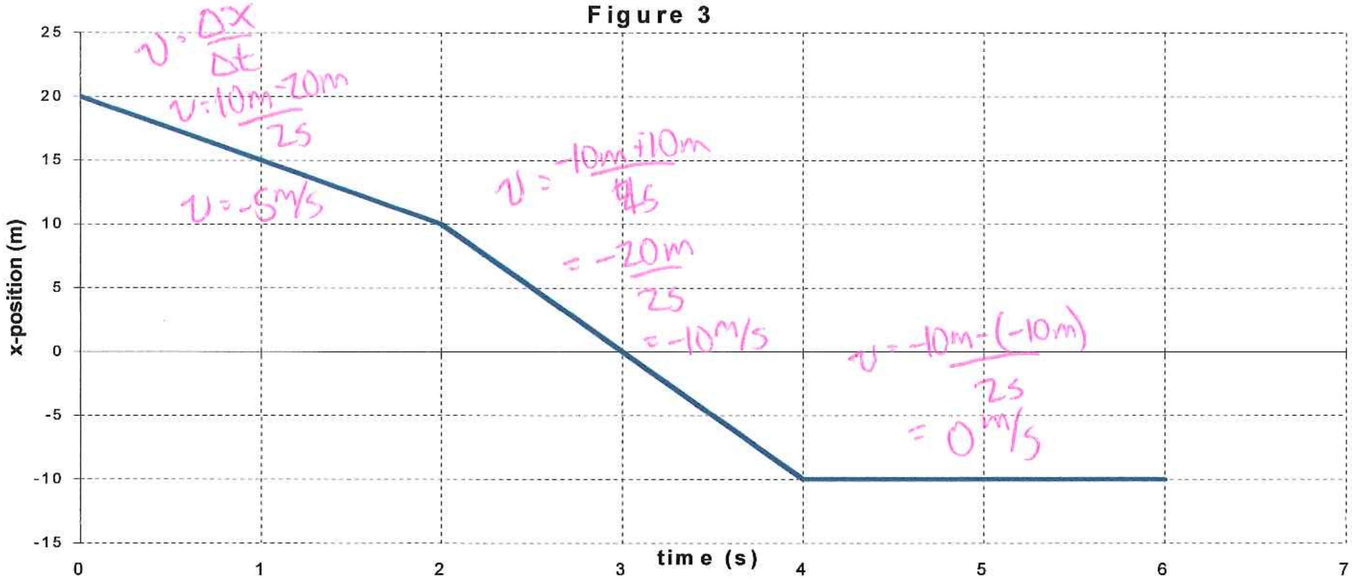


CHECK: AREA = 10(2) = 20

= -10(2) = -20

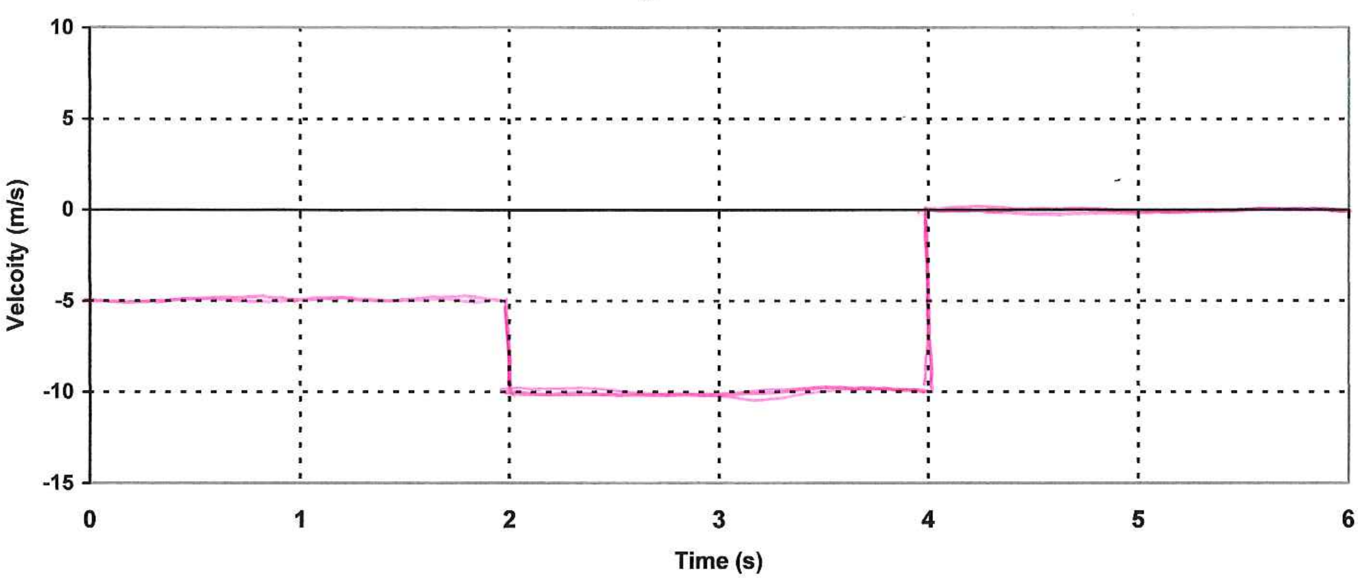
SO CHANGES POSITION BY 20 METERS EVERY TWO SECONDS ✓

Figure 3



5. In Figure 3b sketch the velocity function for a motion in which the position function is given by Figure 3

Figure 3 b



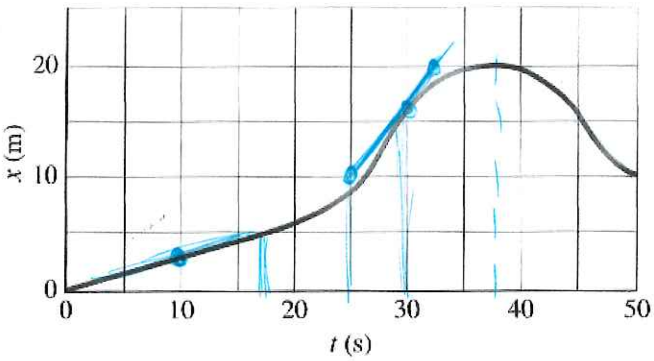


FIGURE 2-26

6. The position of a rabbit along a straight tunnel as a function of time is plotted in Fig. 2-26. What is its instantaneous velocity (a) a t = 10.0 s and (b) at t= 30.0 s? What is its average velocity (c) between t = 0 and t = 5.0 s, (d) between t = 25.0 s and t = 30.0 s, and between t = 40.0 s and t = 50.0 s?

a. $v = \frac{5m - 0m}{17.5s} = \boxed{.2857\ m/s}$

b. $v = \frac{20m - 10m}{7.5s} = \boxed{1.333\ m/s}$

c. $\bar{v} = \frac{\Delta x}{\Delta t} = \frac{2m - 0m}{5s} = \boxed{.4\ m/s}$

d. $\bar{v} = \frac{16m - 8m}{5s} = \boxed{1.6\ m/s}$

e. $\bar{v} = \frac{10m - 20m}{10s} = \boxed{-1\ m/s}$

7. In figure 2-26, (a) during what time periods, if any, is the object's velocity constant? (b) At what time is its velocity the greatest? (c) At what time, if any, is the velocity zero? (d) Does the object run in one direction or in both along its tunnel during the time shown?

a. BETWEEN t = 0 AND t = 18s

b. VELOCITY IS GREATEST WHEN SLOPE IS GREATEST, SO AROUND $\boxed{t = 30sec}$

c. v = 0 WHEN SLOPE = 0 SO AT TIME $\boxed{t = 38\ sec}$

d. BOTH DIRECTIONS, CHANGES VELOCITY FROM POSITIVE TO NEGATIVE SO SWITCHED DIRECTIONS.

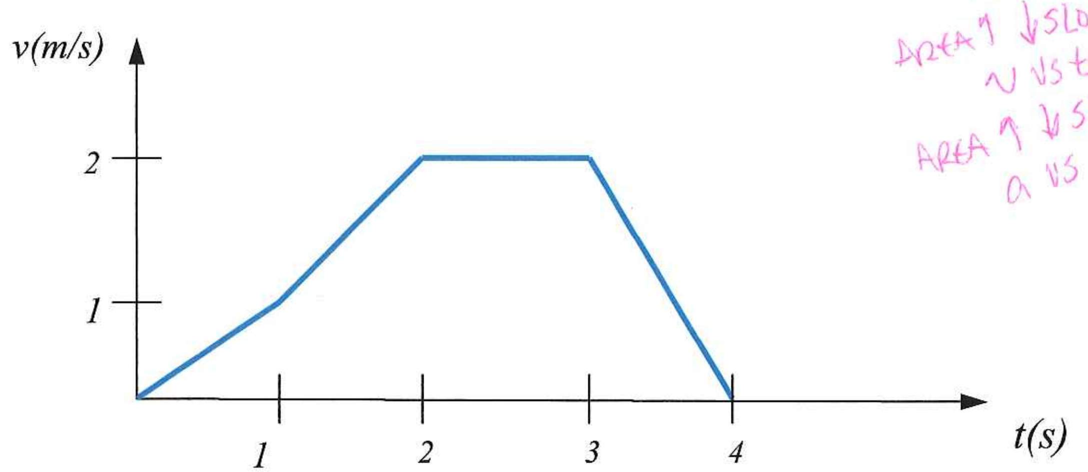
Lesson 5. Velocity vs. Time Graphs

Velocity vs Time graphs are "slope pictures" of the average acceleration equation

$$\boxed{\vec{a} \equiv \frac{\Delta \vec{v}}{\Delta t} = \frac{\text{y-axis}}{\text{x-axis}} = \frac{rise}{run} = slope}$$

SAME AS GOING FROM x VS t TO v VS t

x VS t
AREA ↑ ↓SLOPE
v VS t
AREA ↑ ↓SLOPE
a VS t



Using the graph above calculate the following.

a.) The average acceleration from 0 →1 sec

$\vec{a} = \frac{v_f - v_i}{t}$

$a = \frac{1m - 0m}{1s}$ $\therefore \boxed{\vec{a} = 1 m/s^2 \hat{x}}$

b.) The average acceleration from 1 → 2 sec

$\vec{a} = \frac{2m - 1m}{1s}$ $\Rightarrow \boxed{\vec{a} = 1 m/s^2 \hat{x}}$

c.) The average acceleration from 2 → 3 sec

$\vec{a} = \frac{2 m/s - 2 m/s}{1s}$ $\therefore \boxed{a = 0 m/s^2}$

d.) The average acceleration from 3 → 4 sec

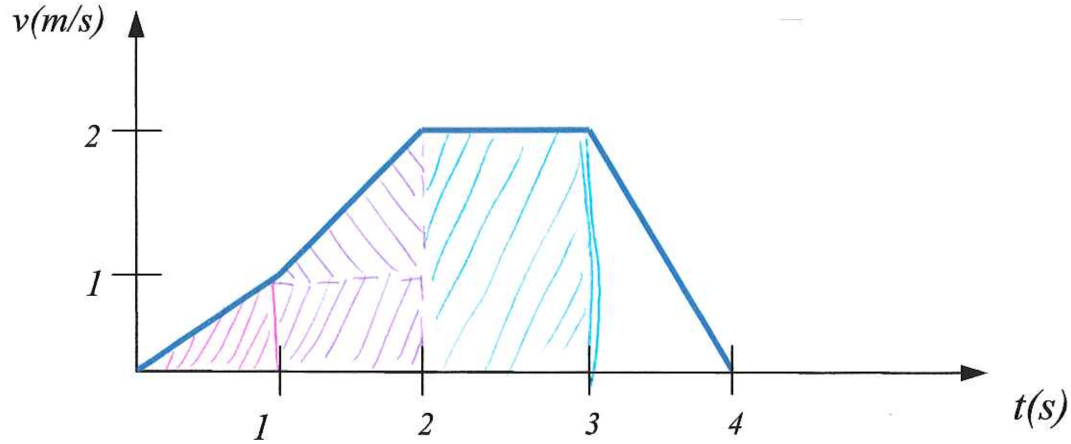
$\vec{a} = \frac{0 m/s - 2 m/s}{1s}$ $\Rightarrow \boxed{\vec{a} = -2 m/s^2 \hat{x}}$

Velocity vs Time graphs are also "area pictures" for a displacement calculation

$$\vec{v} \equiv \frac{\Delta\vec{x}}{\Delta t}$$

$$\Delta\vec{x} = \vec{v}\ \Delta t$$

$$\Delta\vec{x} = (\text{height})\ (\text{base})$$



Using the graph above calculate the following.

e.) The displacement $\Delta\vec{x}$ from 0 →1 sec

$\Delta x = \frac{1}{2}(1)(1) = \boxed{1/2\, m\hat{x}}$

f.) The displacement $\Delta\vec{x}$ from 1 → 2 sec

$\Delta x = 1(1) + \frac{1}{2}(1)(1)$

$= \boxed{3/2\, m\hat{x}}$

g.) The displacement $\Delta\vec{x}$ from 2 → 3 sec

$\Delta x = 1(2) = \boxed{2 m\hat{x}}$

h.) The displacement $\Delta\vec{x}$ from 0 → 3 sec

TOTAL = $1/2 + 3/2 + 2$

$\boxed{\Delta\vec{x} = 4 m\hat{x}}$

Instantaneous Velocity

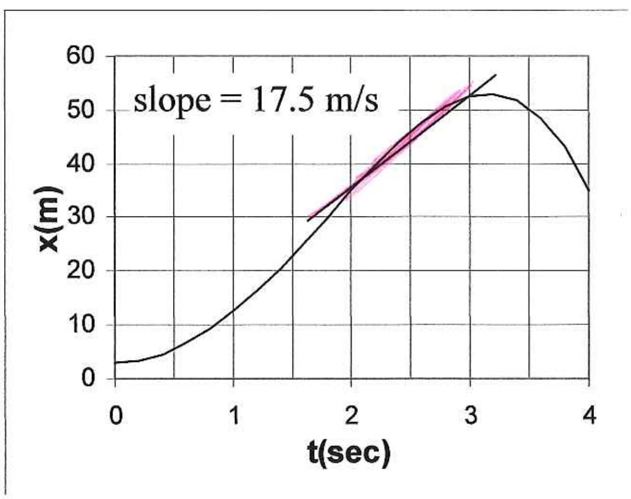
Recall: $\vec{v}_{ave} = \frac{\Delta\vec{x}}{\Delta t} = \frac{x(t_f) - x(t_i)}{t_f - t_i}\,\hat{x}$ (Average velocity)

Consider the function x(t): $x(t) = 3\text{ m} + \left(10\ \frac{m}{s^2}\right) t^2 - \left(0.5\ \frac{m}{s^4}\right) t^4$

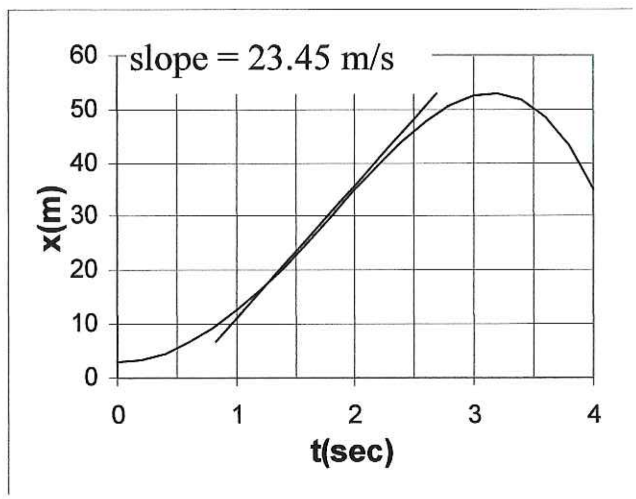
PLUG IN t=3 AND t=2

A. $\vec{v}_{ave} = \frac{52.5\text{ m} - 35.0\text{ m}}{3\text{ s} - 2\text{ s}} = 17.5\frac{m}{s}\,\hat{x}$ ✓ 2 s < t < 3 s Δt = 1 sec

B. $\vec{v}_{ave} = \frac{39.69\text{ m} - 35.0\text{ m}}{2.2\text{ s} - 2\text{ s}} = 23.44\frac{m}{s}\,\hat{x}$ 2 s < t < 2.2 s Δt = 0.2 sec



A.



B.

Δ t (sec)	v_{ave} (m/s)
1	17.5
0.2	23.45
0.01	23.98
0.001	23.998

AS Δt DECREASES, SLOPE GETS CLOSER TO ACTUAL VALUE

Define: **The instantaneous velocity at time t_i is the slope of the line <u>tangent</u> to the curve X(t) at the time t_i.**

The instantaneous velocity at the time $t = t_i$ is the limiting value we get by letting the upper value of the t_f approach t_i. Mathematically this is expressed as:

$$\vec{v}(t) = \frac{dX(t)}{dt} = \lim_{t_f \to t_i}\left[\frac{X(t_f) - X(t_i)}{t_f - t_i}\right]$$

The velocity function $\vec{v}(t)$ is the time derivative of the position function $X(t)$.

Differentiation (Calculus)

Acceleration

When the instantaneous velocity of a particle is changing with time, the particle is accelerating

$$\vec{a}_{ave} = \frac{\Delta\vec{v}}{\Delta t} = \frac{v_f - v_i}{t_f - t_i}\,\hat{x} \qquad \text{(Average Acceleration)}$$

Units: $\vec{a}_{ave} = \frac{m/s}{s} = \frac{m}{s^2}$

Example: If a particle is moving with a velocity in the x-direction given by

$$\vec{v}(t) = \left(3\frac{m}{s^2}\right)t$$

a.) What is the average acceleration over the time interval $6\ s \leq t \leq 12\ s$?

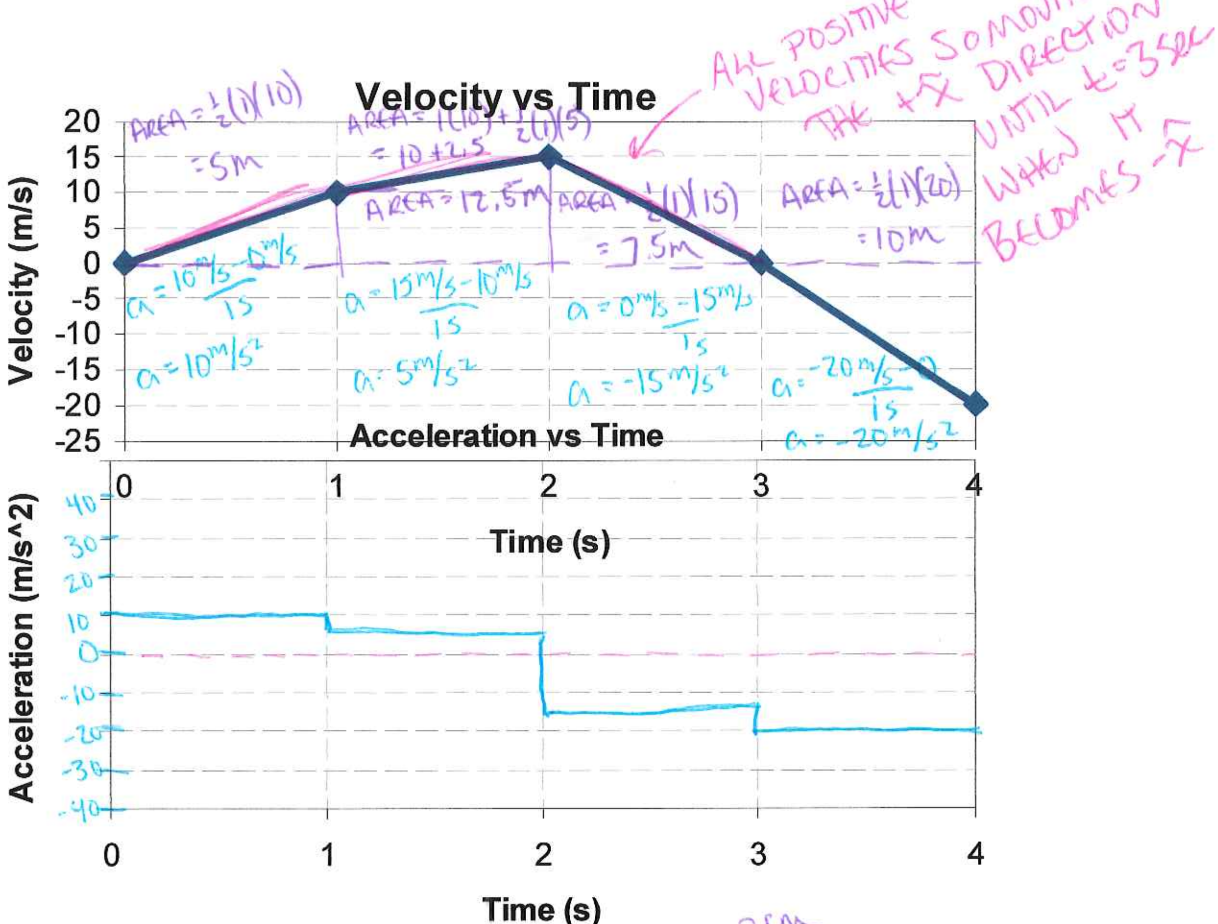
$v(12) = (3\,m/s^2)(12\,s) = 36\,m/s$

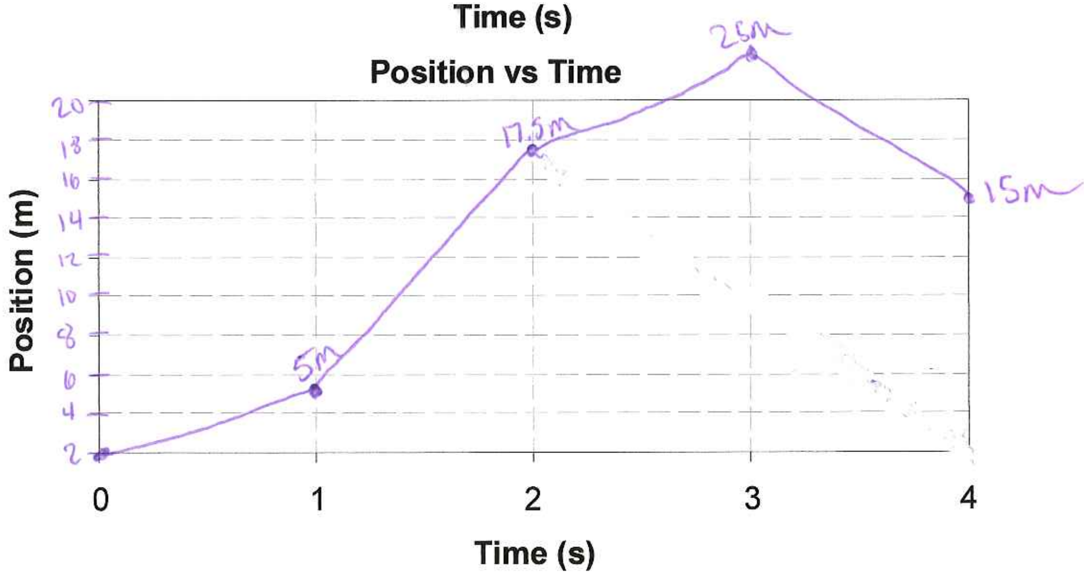
$v(6) = (3\,m/s^2)(6\,s) = 18\,m/s$

$\vec{a}_{AVE} = \frac{v_{12} - v_6}{12 - 6} = \frac{36 - 18}{6} = \boxed{3\,m/s^2\,\hat{x}}$

Lesson 5 Problems: Velocity vs Time Graphs

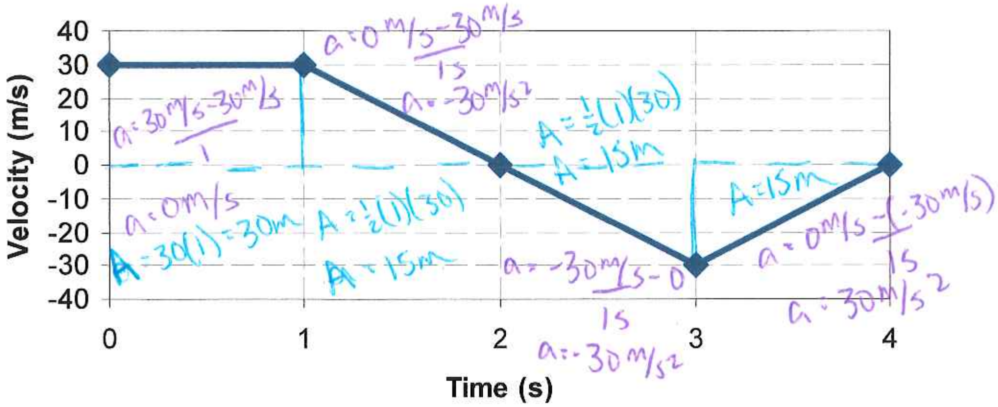
Using the velocity graph below generate the acceleration and the position graph. Assume the particle starts at the origin for the position graph.



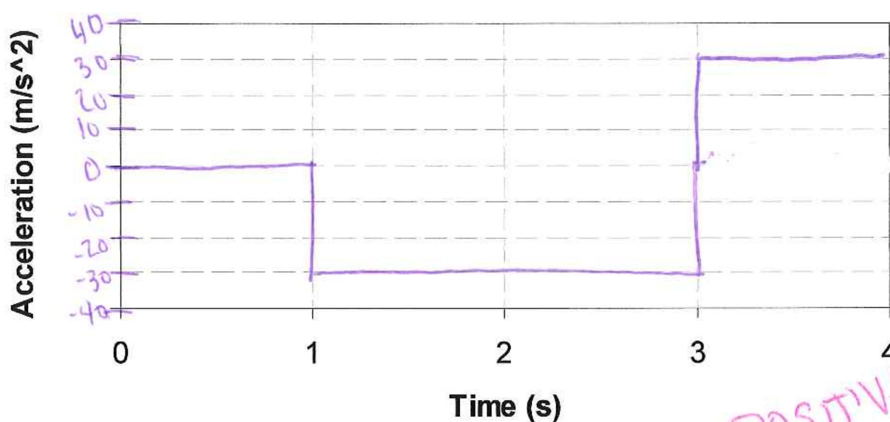


Using the velocity graph below generate the acceleration and the position graph. Assume the particle starts at the origin for the position graph.

Velocity vs Time

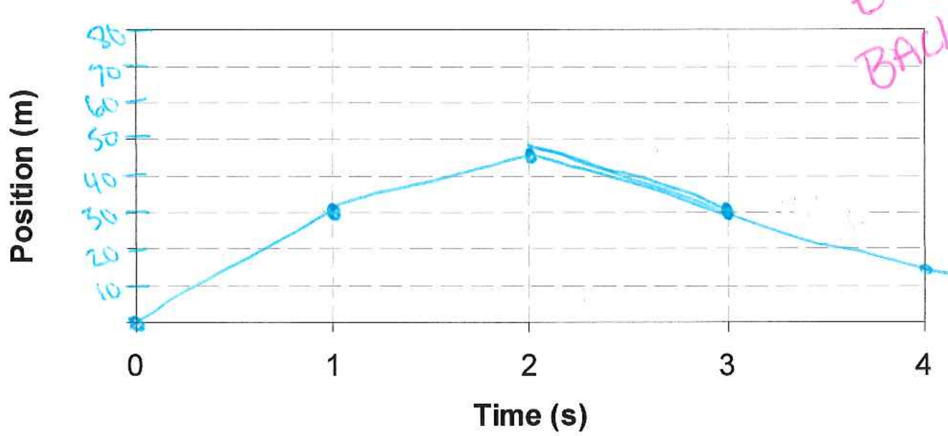


Acceleration vs Time



POSITIVE VELOCITY = $+\hat{x}$ DIRECTION. STOPS AT $t=2$ AND MOVES BACK IN $-\hat{x}$ DIRECTION

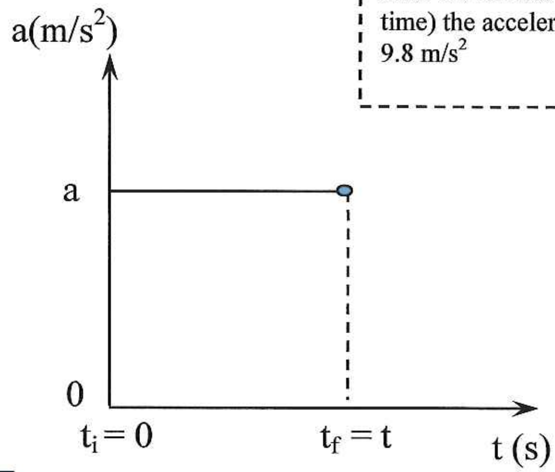
Position vs Time



Lesson 6: Special Case: Constant Acceleration

$a_g = -9.8\,m/s^2\,\hat{y}$

> We make the assumption that the acceleration does not change. Near the surface of the earth, (where most of us spend most of our time) the acceleration due to gravity is approximately constant $a_g = 9.8\ m/s^2$



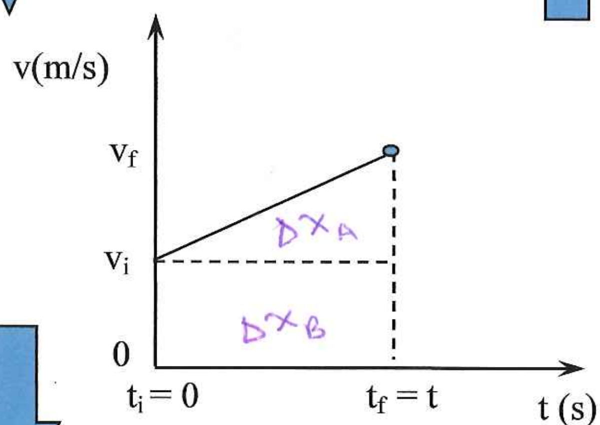
$a = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - \cancel{t_i}^{\,0}} = \frac{v_f - v_i}{t}$

$at = v_f - v_i$

$v_f = at + v_i$ ✓

$$\boxed{v_f = v_i + a\,t} \quad 1.$$

Area! — *Slope!* $\cancel{y = b + mx}$



Δx = AREA BELOW v FUNCTION

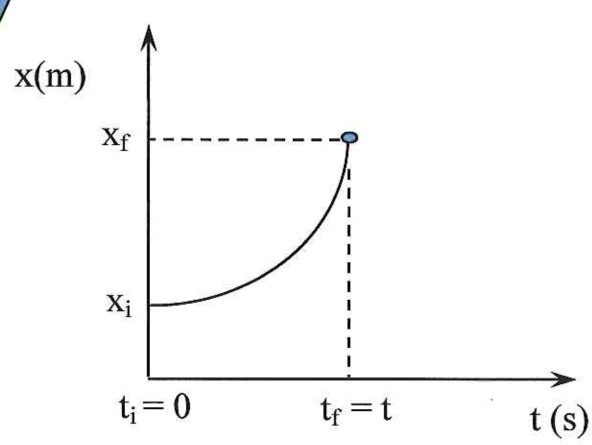
$= \Delta x_A + \Delta x_B$

$= \frac{1}{2}(t)(v_f - v_i) + v_i t$

$= \frac{1}{2}(t)(\cancel{v_i} + at - \cancel{v_i}) + v_i t$

$\Delta x = \frac{1}{2}at^2 + v_i t$ ✓

Area! — *Slope!*



$$\boxed{x_f = x_i + v_i t + \frac{1}{2}a\,t^2} \quad 2.$$

Solving for the 3rd constant acceleration equation

Solve equation 1 for t and substitute t into equation 2 to get the following equation.

$$\boxed{v_f^2 = v_i^2 + 2\,a\,\Delta x} \quad \textbf{3.}$$

$$v_f = v_i + at$$

$$v_f - v_i = at$$

$$t = \frac{v_f - v_i}{a}$$

$$\Delta x = \frac{1}{2}at^2 + v_i t$$

$$\Delta x = \frac{1}{2}\cancel{a}\left(\frac{v_f - v_i}{\cancel{a}}\right)^2 + v_i\left(\frac{v_f - v_i}{a}\right)$$

$$\Delta x = \frac{1}{2}\cancel{a}\,\frac{(v_f^2 - 2v_i v_f + v_i^2)}{a^2} + \frac{v_i v_f - v_i^2}{a}$$

$$2a\Delta x = v_f^2 - \cancel{2v_i v_f} + v_i^2 + \cancel{2v_i v_f} - 2v_i^2$$

$$2a\Delta x = v_f^2 - v_i^2$$

$$\therefore v_f^2 = v_i^2 + 2a\Delta x \quad ✓$$

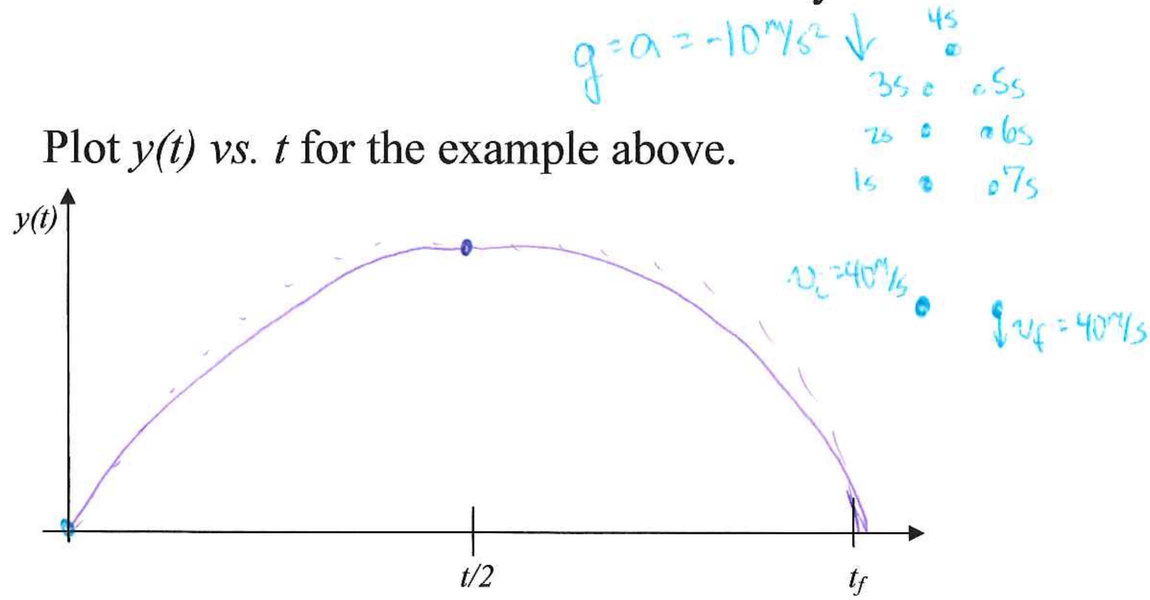
FREE-FALL ACCELERATION

($9.8\ m/s^2 = 32\ ft/s^2$)

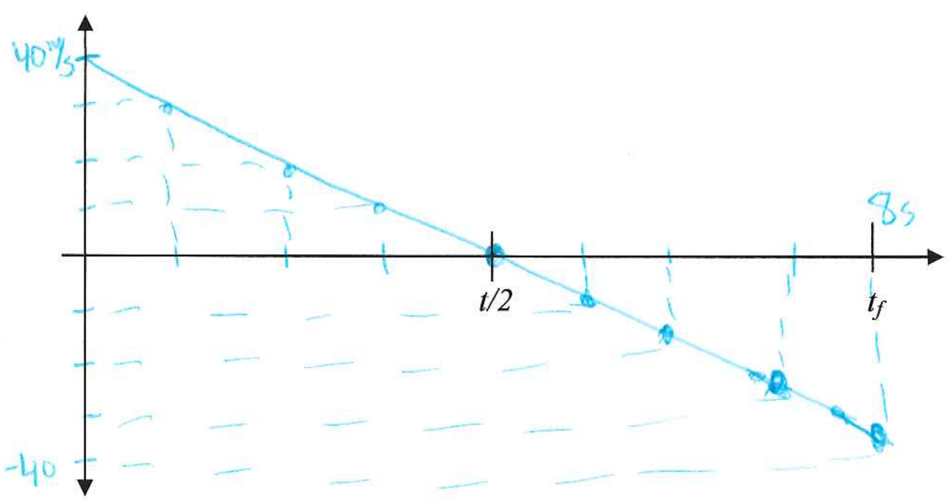
Consider a ball is thrown straight up.
It is in "Free Fall" the moment it leaves you hand.

$g = a = -10\ m/s^2 \downarrow$

Plot *y(t) vs. t* for the example above.

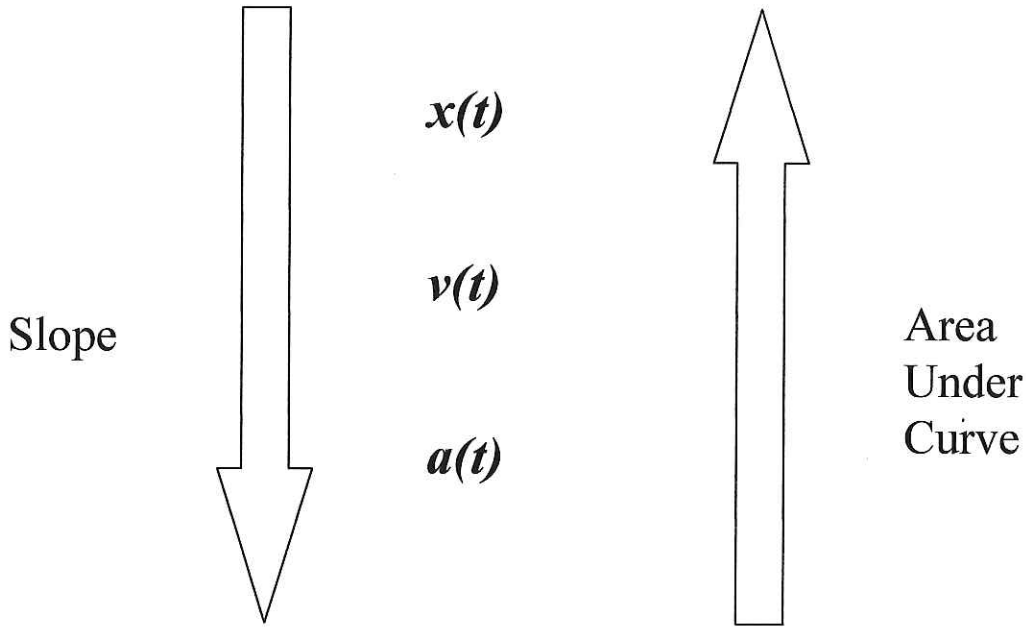


Plot *v(t) vs. t*



FINAL NOTES ON CH 2.

Remember, when going between the following graphs



Problem Solving with the constant acceleration equations

1. Write down all three equations in the margin
2. $a = -\,9.8\ \mathrm{m/s^2}$ for free fall problems
3. Analyze the problem in terms of *initial* and *final* sections.

Lesson 6 Problems: Constant Acceleration Problems

1. A sports car accelerates from rest to 95 km/h in 6.2 s. What is its average acceleration in m/s^2?

$v_f = 95\ km/hr$

$v_i = 0\ km/hr$ $\qquad v_f = v_i + at$

$t = 6.25 \times \frac{1\ hr}{3600\ s} = .0017\ hr$ $\qquad 95\ km/hr = 0\ km/hr + a(.001722\ hr)$

$a = ?$ $\qquad a = 55161.29\ km/hr^2$

$$\frac{55161.29\ km}{hr} \times \frac{1}{hr} \times \frac{1\ hr}{3600\ s} \times \frac{1\ hr}{3600\ s} \times \frac{1000\ m}{1\ km} = \boxed{4.256\ m/s}$$

2. A sports car is advertised to be able to stop in a distance of 50 m from a speed of 90 km/h. What is its acceleration in m/s^2? How many "g's" is this (g = 9.80 m/s^2)?

$\Delta x = 50m$ $\qquad v_f^2 = v_i^2 + 2a\Delta x$

$v_i = 90\ km/hr = 90{,}000\ m/hr$ $\qquad (0\ km/hr)^2 = (90{,}000\ m/hr)^2 + 2a(50m)$

$v_f = 0\ km/hr$ $\qquad a = -\frac{81000000\ m}{hr^2} \times \frac{1\ hr}{3600\ s} \times \frac{1\ hr}{3600\ s} = -6.25\ m/s^2$

$a = ?$

3. A car accelerates from 12 m/s to 25 m/s in 6.0 s. What was its acceleration? How far did it travel in this time? Assume constant acceleration

$v_i = 12\ m/s$ $\qquad v_f = v_i + at$

$v_f = 25\ m/s$ $\qquad 25\ m/s = 12\ m/s + a(6s)$

$t = 6.0s$ $\qquad \boxed{a = 2\tfrac{1}{6}\ m/s^2}$

$a = ?$ $\qquad \Delta x = v_i t + \frac{1}{2}at^2$

$\Delta x = ?$ $\qquad = 12\ m/s(6s) + \frac{1}{2}(2\tfrac{1}{6}\ m/s^2)(6s)^2$

$= 72m + 39m$

$\boxed{\Delta x = 111m}$

4. A car traveling at 90 km/h strikes a tree. He front end of the car compresses and the driver comes to rest after traveling 0.80 m. What was the average acceleration of the driver during the collision? Express the answer in terms of "g's" where g = 9.80 m/s^2.

$v_i = 90\,\text{Km/hr} \times 1\,\text{hr}/3600\,\text{s} \times 1000\,\text{m}/1\,\text{Km} = 25\,\text{m/s}$

$v_f = 0\,\text{m/s}$

$\Delta x = .80\,\text{m}$

$a = ?$

$v_f^2 = v_i^2 + 2a\Delta x$

$(0\,\text{m/s})^2 = (25\,\text{m/s})^2 + 2a(.80\,\text{m})$

$a = -390.625\,\text{m/s}^2$

$\boxed{a = 39.859g}$

5. A speeding motorist traveling 120 km/h passes a stationary police officer. The officer immediately begins pursuit at a constant acceleration of 10 km/h/s (note the mixed units). How much time will it take for the police officer to reach the speeder, assuming that the speeder maintains a constant speed? How fast will the police officer be traveling at this time?

MOTORIST: $120\,\text{Km/hr}$

OFFICER: $a = \frac{10\,\text{Km}}{\text{hr}\cdot\text{s}} \times \frac{3600\,\text{s}}{1\,\text{hr}} = 36000\,\text{Km/hr}^2$

$v_i = 0\,\text{Km/hr}$

$v = d/t$

$120\,\text{Km/hr} = d/t$

$\Delta x = \cancel{v_i t}^{0} + \frac{1}{2}at^2$

$\Delta x = \frac{1}{2}(36000\,\text{Km/hr}^2)\,t^2$

$120\,\text{Km/hr} = \frac{\left(\frac{1}{2}(36000\,\text{Km/hr}^2)t^{\cancel{2}}\right)}{\cancel{t}}$

$t = .00\overline{666}\,\text{hr} \times \frac{3600\,\text{s}}{1\,\text{hr}} = \boxed{24\,\text{s}}$

$v_f = v_i + at$

$v_f = 0\,\text{Km/hr} + (36{,}000\,\text{Km/hr}^2)(.00\overline{666}\,\text{hr})$

$v_f = \frac{240\,\text{Km}}{\text{hr}} \times \frac{1000\,\text{m}}{1\,\text{Km}} \times \frac{1\,\text{mi}}{1600\,\text{m}} = \boxed{150\,\text{mph}}$

6. Calculate (a) how long it took King Kong to fall straight down from the top of the Empire State Building (380 m high), and (b) his velocity just before "landing"?

$v_i = 0 m/s$

$\Delta x = -380 m$

$a = -9.8 m/s^2$

$t = ?$

$v_f = ?$

$\Delta x = v_i t + \frac{1}{2}at^2$

$-380 m = \frac{1}{2}(-9.8 m/s^2)t^2$

$\boxed{t = 8.806 s}$

$v_f = v_i + at$

$= 0 m/s + (-9.8 m/s^2)(8.806 s)$

$\boxed{v_f = -86.298 m/s}$

7. A foul ball is hit straight up into the air with a speed of about 25 m/s. (a) How high does the ball go? (b) How long is it in the air?

$v_i = 25 m/s$

$a = -9.8 m/s^2$

$v_f = 0 m/s$

$\Delta x = ?$

$t = ?$

$v_f^2 = v_i^2 + 2a\Delta x$

$(0 m/s)^2 = (25 m/s)^2 + 2(-9.8 m/s^2)\Delta x$

$\boxed{\Delta x = 31.89 m}$

$v_f = v_i + at$

$0 m/s = 25 m/s + (-9.8 m/s^2)t$

$t = 2.55 s$ TO GO UP SO TOTAL $\boxed{t = 5.102 s}$

8. A helicopter is ascending vertically with a speed of 5.50 m/s. At a height of 105 m above the Earth, a package is dropped from a windows. How much time does it take for the package to reach the ground?

$\Delta x = -105 m$

$a = -9.8 m/s^2$

$v_i = 0 m/s$

$t = ?$

$\Delta x = v_i t + \frac{1}{2}at^2$

$-105 m = (0 m/s)t + \frac{1}{2}(-9.8 m/s^2)t^2$

$\boxed{t = 4.629 s}$

9. A stone is thrown vertically upward with a speed of 20.0 m/s. (a) How fast is it moving when it reaches a height of 12.0m? (b) How long is required to reach this height? (c) Why are there two answers to (b)?

$v_i = 20.0\ m/s$

$a = -9.8\ m/s^2$

$\Delta x = 12.0m$

$v_f = ?$

$t = ?$

a. $v_f^2 = v_i^2 + 2a\Delta x$

$v_f^2 = (20\ m/s)^2 + 2(-9.8\ m/s^2)(12.0m)$

$v_f^2 = 164.8 \Rightarrow \boxed{v_f = 12.837\ m/s}$

$\Delta x = v_i t + \frac{1}{2}at^2$

$12m = (20\ m/s)t + \frac{1}{2}(-9.8\ m/s^2)t^2$

$0 = -4.9t^2 + 20t - 12$

$t = \frac{-20 \pm \sqrt{20^2 - 4(-4.9)(-12)}}{2(-4.9)} = \frac{-20 \pm 12.837}{-9.8}$

$\boxed{t = .7309,\ 3.3507s}$

10. A person who is properly constrained by an over the shoulder seat belt has a good chance of surviving a car collision if the deceleration does not exceed 30 g's (1g = 9.8 m/s^2). Assuming uniform deceleration of this value, calculate the distance over which the front end must collapse if a crash brings the car to rests from 100 km/hr.

$v_i = 100\ km/hr \times \frac{1000m}{1km} \times \frac{1hr}{3600} = 27.\overline{77}\ m/s$

$v_f = 0\ km/hr$

$a = 30g = -294\ m/s^2$

$\Delta x = ?$

$v_f^2 = v_i^2 + 2a\Delta x$

$(0\ m/s)^2 = (27.77\ m/s)^2 + 2(-294\ m/s^2)\Delta x$

$\boxed{\Delta x = 1.31m}$